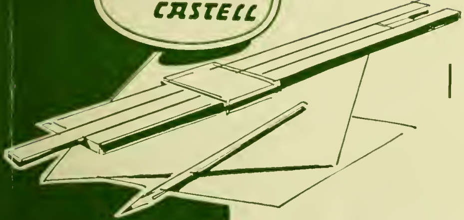
FABER-CASTELL



INSTRUCTIONS

CASTELL PRECISION
SLIDE RULES
„System Darmstadt"

No. 67/54 b 67/54 R

No. 1/54 111/54 111/54 A

No. 4/54

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Note

The Slide Rule **"System Damstadt"**
resulted from the work of the
Mathematical Institute of the Technical University of Darmstadt
under the direction of Professor W a l t h e r
and was introduced by the firm A. W. Faber-Castell at the instance of Professor Walther.

Description of the Slide Rule

The Slide Rule **System Darmstadt** is a general purpose slide rule. Its logarithmic scales make possible all the calculations which are met with in mathematics and their practice. It carries no special scales such as would be required for commercial, nautical purpose, reinforced concrete, or any other narrow field of activity.

The scales of the System Darmstadt Slide Rules are grouped as follows:

1. The **Main Scal**es A, B, C, D (x) and CI.

2. The **Supplementary Scal**es K, P ($\sqrt{1-x^2}$), the trigonometrical scales, and the log-log scales LL_1, LL_2, LL_3.

The Main Scales.

Even the simplest general slide rule has the upper scales, **A** and **B**, and the lower scales, **C** and **D**. (Fig. 1)

In addition to these four scales, there is a **reciprocal,** or reversed C, scale on the centre of the slide between B and C. This scale, **CI**, runs from **10 to 1** (Fig. 1).

These five scales are extended a short distance at each end, the extra graduations being a different colour to the main part of the scales.

For all calculations containing only multiplication and division, squares and square roots the three scales C, D, and CI should be used.

The Supplementary Scales

Additional scales are provided to facilitate calculations other than multiplication, division, squares and square roots:

View of the 10 inch Slide Rule No. 1/54

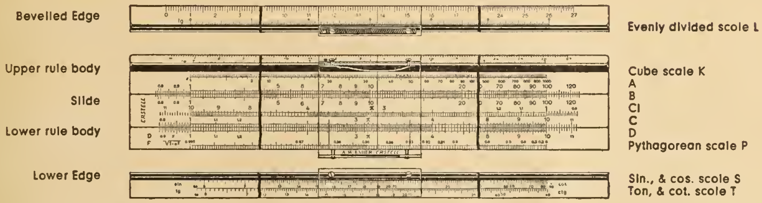


Fig. 1

The **evenly divided scale L** on the upper part of the rule body is used in conjunction with Scale D for reading common logarithms.

The **cube scale K** is on the rule face above A. It is graduated from **1 to 1000,** and is used with Scale D.

The **Pythagorean scale P** ($\sqrt{1-x^2}$) is on the rule face below D, with which scale it is to be employed.

The **trigonometrical scales S and T** will be found on the lower part of the rule body.

Finally, there is a **log-log-scale**, graduated in three sections, LL_1, LL_2, LL_3, **from 1·01 to 10^5,** on the back of the slide.

The Cursor

enables these scales to be employed in any combination. The long centre line is generally used, while the four short lines at the sides are provided for a special purpose which will be explained later. (p. 15)

The Decimal Point

As the upper scales, **A** and **B**, run from 1 to 100, and the lower from 1 to 10, a novice is inclined to think that it is only possible to use the slide rule for numbers within these limits. This is not so, since the position of the decimal point is ignored in slide rule working. For instance, to multiply 320 by 580, it would be possible to multiply 3-2 by 5-8 and increase the answer then thousand times, or in other words, move the decimal point four places to the right. The graduation 3 on any of the scales may be taken to represent 30, 300, 3,000, etc., or 0·3, 0·03, 0·003, etc. In slide rule working significant figures only are considered, and the position of the decimal point is found from a rough estimate of the size of the answer. In practical problems the number of figures is obvious.

Reading the Scales

Slide Rules with graduated length 10 inch. Nos. 1/54, 111/54, 111/54 A
Subdivision 1 to 2

Let us first turn our attention to the lower scales, **C** and **D.** Here it should be noted that the tenths are shown and numbered between **1 and 2**, these tenths in their turn being subdivided in a like manner (hundredths). The division-marks are thus read off as follows from the start: 100-101-102-103 . . . . 109-110-111-112-113 . . . . 198-199-200.

Exercises: Set the cursor with its hair line over the following values: 175, 163, 157, 130, 103, 170, 107, 111, 191.

When calculating with the slide rule, however, we are not merely concerned with numbers shown by a mark on the scales. We must also be able to set the cursor-line correctly to further imaginary subdivisions in between the hundredths (that is, to thousandths). This is by no means so difficult as it first appears. Firstly practice finding the exact centre.

Exercises: Set the cursor-line to 1075, 1355, 1675, 1425, 1985, 1705, 1075.

For all other values it is advisable to work from the centre-position outwards, so that if the cursor has to be set, for example, to 1074, it should first be placed so that its line is in the right position for 1075 and then moved slightly to the left. If the value desired is 1688, on setarts at 1690 and then goes back a little way. After a certain amount of practice the positions of the tenths can be satisfactorily estimated.

The setting of the cursor-line between two adjacent division-marks is an operation which the user must practice regularly. The best way of doing this is to draw two vertical lines at a distance of about 10 mm. and then placing a thread on each tenth of the distance in turn, judging the position with the eye in each case. One can then use a ruler to check ones accuracy of judgment.

Exercises: Place cursor-line over 1172, 1784, 1098, 1346, 1777, 1007, 1703.

Finally, set the cursor-line somewhere between 1 and 2 and read off the number denoted by that position.

Subdivision from 2 to 4.

Now let us consider our next division, that extending from **2 to 4**. Here we first find the tenths marked, as before, but the only further divisions marked are the fifths. The values from 2 onwards are: 200-202-204-206-208-210-212 . . . 396-398-400. In setting the odd hundredths, therefore, the position must be judged with the eye.

Exercises: Place cursor-line over the values 207, 347, 277, 209, 315, 373.

In this section it is recommended that the beginner should not for the moment attempt to set the cursor to thousandths. If the requires 2358, for instance, he should round it off to 236, 2073 being rounded off to 207, and so forth.

Finally, the cursor is placed at any desired point between 2 and 4, the user endeavouring to take an accurate reading of the result.

Subdivision from 4 to 10.

From **4** onwards to the end of the lower scales, only the halves are marked between the tenths. After 4, therefore, readings are taken as follows: 405-410-415 etc., up to 995-1000. All the other hundredths must be judged with the eye. First place the cursor-line over the following easier numbers: 4225; 7875; 9175; 6025, etc. If the setting required is 423, the best method is to begin with 4225 and then to move the cursor-line a little towards the right. For 787 one starts with 7875, then moving a little to the left.

Exercises: Set to 633; 752; 927; 538; 467.

For 444 and 446 one starts with 445 and then moves to the left or to the right respectively. On the same principle one starts at 790 for 789 or 791.

Exercises: Set to 908; 426; 709; 627; 517.

The user is also advised to select numbers of his own for setting and reading.

Slide Rules with graduated length 20 inch. No. 4/54

If one is using a slide rule with a graduated length of 20 inch the subdivision is naturally different. It is explained briefly below: —

The large slide rules with a graduated length of 20 inch are divided up as follows: on **C** and **D**, between 1 and 2, we first find the tenths, then tenths of the latter (hundredths) and finally the halves of these further tenths, i.e. the two-hundredths. Our readings are thus: 1005-1010-1015-1020 etc. up to 1990-1995 and 2000. From 2 to 5 the tenths are marked and then further subdivided into tenths (i. e. hundredths). Our readings are thus: 201-202-203 to 498-499 and 500. From this point onwards to the end of the scales the tenths are marked, with fifths in between them (fiftieths). The readings are therefore: 502-504-506 etc. up to 996-998 and 1000.

On the upper scales of the large slide rules the interval between 1 and 2 is subdivided in the same way as that between 2 and 5 on the lower scales, the subdivision of the interval between 2 and 4 corresponding to that between 5 and 10 below, while in the interval between 4 and 10 only the halves of these subdivisions, i.e. the twentieths, are marked, so that the readings are: 405-410-415 etc. op to 990-995-1000.

Slide Rules with graduated length 5 inch. Nos. 67/54 b and 67/54 R

On the small slide rules with a graduated length of 5 inch. only the tenths, with the fifths of the latter, i.e. the fiftieths, are marked between 1 and 2. Readings are thus taken from 102—104—106 to 196—198 and 200. From 2 to 5, we find only the tenths, with their respective halves, so that readings are taken from 205—210—215 etc. to 490—495 and 500. From 5 to 10 only the tenths are marked. On the upper scales the subdivision in the interval from 1 to 3 is the same as that between 2 and 5 on the lower scales, whilst that in the interval between 3 and 6 is the same as that between 5 and 10 below. In the interval from 6 to 10 only the fifths are marked. The reading in this case is thus from 62—64 to 96—98 and 100.

How To Calculate With The Slide Rule

As the scales of the slide rule are tables of logarithms, their operation is based on logarithmic laws. It is well known that:

1. **Multiplication** of two factors is carried out by the **addition** of their logarithms.
2. **Division** is carried out by **subtracting** the logarithm of the divisor from the logarithm of the dividend.

The table of logarithms, therefore, replaces every method of calculating by a simpler operation, and the slide rule even avoids these simple operations, since they are graphically carried out. It follows, therefore, that on the slide rule **Multiplication of two numbers is transformed into addition of two lengths.**

Division of one number by another is changed to subtraction of one length from another.

Multiplication

Example: 2·5 × 3 = 7·5 (Fig. 2).

$\boxed{a \times b}$

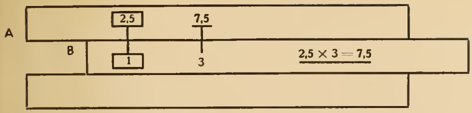


Fig. 2

The left index line 1 of the slide (Scale **B**) is placed under the 2·5 of the upper "body" scale (**A** 25), the cursor line then being placed above the 3 of the upper slide scale (**B** 3). The product (7·5) can then be read off beneath the cursor line on the upper "body" scale (**A** 75). Exactly the same procedure can be adopted on the lower scales, and here you get more accurate results.

Example: 2·45 × 3 = 7·35 (Fig. 3).

$\boxed{a \times b}$

Fig. 3

The 1 on the slide (**C** 1) is placed above the 2·45 on the lower "body" scale (**D** 245), the cursor line then being placed above the 3 on the lower slide scale (**C** 3). The product (7·35) can then be read off underneath the cursor line on the lower "body" scale (**D** 735).

Example: 7·5 × 4·8 = 36 (Fig. 4).

$\boxed{a \times b}$

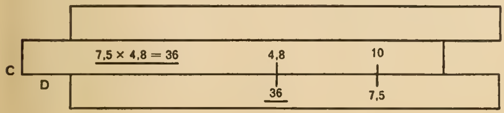


Fig. 4

When calculations are carried out on the lower scales, it will be found that the second factor of a multiplication problem sometimes cannot be selected within the scope of the lower "body" scale. In this case, **C** 10 is placed above the first factor, the cursor line then being placed above the second, after which the result can be read off, as before, beneath the cursor line.

Division

E x a m p l e : 9·85 ÷ 2·5 = 3·94

$$\boxed{\frac{a}{b}}$$

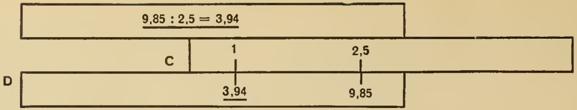


Fig. 5

The denominator 2·5 on the lower slide scale (**C** 25) is placed above the numerator 9·85 on the lower „body" scale (**D** 985) and the quotient (3·94) is read off underneath the beginning of the slide (**C** 1).

This calculatory process can naturally be performed on the upper scales likewise. The result is read off above the right-hand or left-hand extremity of the slide (**B** 1 or **B** 100) on Scale **A.**

How to Form Tables

E x a m p l e : To convert yards into metres. (82 yds. equal 75 m.)

$$\boxed{\frac{a}{b} \div \frac{c}{d} \div \frac{e}{f}}$$

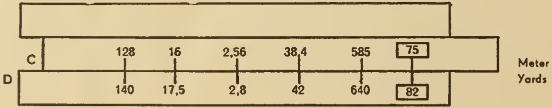


Fig. 6

Place **C** 75 over **D** 82. This automatically produces a comparative Table, from which the following readings can be taken: 42 yds. are 38·4 m.; 2·8 yds. are 2·56 m.; 640 yds. are 585 m.; 16 m. are 17·5 yds.; 128 m. are 140 yds., etc.

Compound calculations

Multiplications and divisions in immediate sequence can easily be made with the Calculating Rule. The intermediate results need not be read off if it is not necessary to know them, and, after the last setting, the correct final result will appear. It is best to begin such calculations with a division, then follow with a multiplication, then another division and again a multiplication and so on.

$$\boxed{\frac{a \times b \times c}{d \times e \times f}}$$

Example: $\dfrac{13{\cdot}8 \times 24{\cdot}5 \times 3{\cdot}75}{17{\cdot}6 \times 29{\cdot}6 \times 4{\cdot}96}$

We start by dividing 13·8 by 17·6. Therefore we place **D** 138 and **C** 176 one under the other. Do not read off the answer — approximately 0·8 — but multiply it immediately by 24·5, by placing the cursor-line on **C** 245. Similarly, no reading is taken of the answer — about 19 — and it is simply divided by 29·6. For this purpose, keep the cursor-line firmly in its position and slide **C** 296 under it. Once again, the result (0·65) is not "read" but multiplied at once by 3·75, this being done by placing the cursor-line on **C** 375. The result is merely "retained" by the cursor-line, as before, and divided by 4·96, by sliding **C** 496 under the cursor-line. Only then do we read off the figures of the final answer, 491, above under **C** 10 — and our rough calculation shows us that the actual answer is 0·491.

The Reciprocal Scale CI

1. In order to find the reciprocal value 1 ÷ a for any given number a, find a on **C** (or **CI**) and read above it on **CI** (or below it on **C**) the reciprocal value. Reading off is done therefore without any movement of the slide and entirely by setting the cursor line.

   Examples: 1 ÷ 8 = 0·125; 1 ÷ 5 = 0·2; 1 ÷ 4 = 0·25; 1 ÷ 3 = 0·333

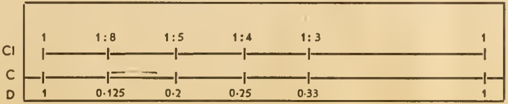


Fig. 7

$$\boxed{\frac{1}{a}}$$

2. To find $1 \div a^2$ move the cursor to a on scale **CI** and read above it on **B** the result.

$\frac{1}{a^2}$

Example: $1 \div 2{\cdot}44^2 = 0{\cdot}168$.
Estimated answer — less than $\frac{1}{5} = 0{\cdot}2$.

Example: Find the resistance R of an appliance having a power of 1320 Watts and drawing a current of 6 A.

Solution: $R = P \times \frac{1}{J^2} = 1320 \times \frac{1}{6^2} = 36{\cdot}7\ \Omega$

3. To find $1 \div \sqrt{a}$, set the cursor line to a on scale **B** and find below it on **CI** the result.

$\frac{1}{\sqrt{a}}$

Example: $1 \div \sqrt{27{\cdot}5} = 0{\cdot}191$.
Estimated answer — less than $\frac{1}{5} = 0{\cdot}2$.

Example: Change a single phase alternating current of 120 V into direct current by means of a transformer.

Solution: $V_d = \frac{2V}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 2V = \frac{1}{\sqrt{2}} \times 240$

Place B 1 under A 2. Result above D 240 = 169·6 V or B 2 above D 240, under C 1 = 169·6 V.

4. To find $1 \div a^3$, set the cursor line over a on scale **CI** and the answer will be found under the cursor line on scale **K.**

$\frac{1}{a^3}$

Example: $1 \div 2{\cdot}26^3 = 0{\cdot}0866$.
Estimated answer — less than $\frac{1}{8} = 0{\cdot}125$.

5. To find $1 \div \sqrt[3]{a}$, move the cursor line to a on scale **K** and read the answer under the cursor line on **CI**.

$$\boxed{\frac{1}{\sqrt[3]{a}}}$$

Example: $1 \div \sqrt[3]{13} = 0{\cdot}425$.

Estimated answer — less than $\frac{1}{2} = 0{\cdot}5$.

6. **Products of three factors** can generally be reached with one setting of the slide. One sets, by means of the cursor, the first two factors against each other on **D** and **CI** respectively, moves the cursor to the third factor on **C** and reads below it on **D**, the final product.

$$\boxed{a \times b \times c}$$

Example: $0{\cdot}66 \times 20{\cdot}25 \times 2{\cdot}38 = 31{\cdot}8$.

Estimated answer — more than $0{\cdot}6 \times 20 \times 2{\cdot}5 = 30$.

Example: What is the area of an ellipse with semi-axes of 15·4 inches and 6·2 inches?

Solution: $A = a\,b\,\pi = 15{\cdot}4 \times 6{\cdot}2 \times 3{\cdot}14 = 300$ square inches.

7. **Compound multiplication and division**

Example: $\dfrac{36{\cdot}4}{3{\cdot}2 \times 4{\cdot}6} = 2{\cdot}472$

One sets by means of the cursor the figures 3-6-4 on **D** against 3-2 on **C**. It is not necessary to read the Intermediary result. Move the cursor line over 4-6 on scale **CI**, which is the same as multiplying by $\frac{1}{4{\cdot}6}$ (= reciprocal value $\frac{1}{c}$) The result of 2·472 is then found under the cursor line on scale **D.**

Example: An alternating current motor of 220 V has a power input of 2860 W with a current of 16 A; find the power factor (cos φ).

$$\boxed{\frac{a}{b \times c}}$$

Solution: $\cos\varphi = \dfrac{P}{V \cdot J} = \dfrac{2860}{220 \times 16} = \mathbf{0.812}$ $(\varphi = 35{,}7°)$

Squares And Square Roots

Both the upper scales are graduated to half length. The change over from **D** to **A** (or from **C** to **B**) gives the **square** of the number to which **D** (or **C**) has been set. **Square roots** are extracted by reversing this procedure (Fig. 8).

$\boxed{a^2}$

Example:
Given the side of a square (47 inches).
Find the area.

$$A = 47^2 = 2209 \text{ sq. in.}$$

$\boxed{\sqrt{b}}$

Example:
What is the diameter of a shaft if P = 50 HP and V = 400 r.p.m.?

$$d = 12 \times \sqrt[4]{\frac{P}{V}} = 12 \times \sqrt{\sqrt{\frac{50}{400}}} = 7{,}138$$

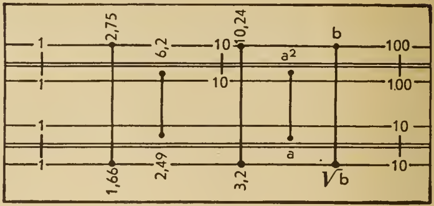


Fig. 8

In extracting a square root it is essential to set the number on the correct half of the upper scale for $\sqrt{x}$ and $\sqrt{10x}$ do not differ merely in the position of the decimal point. If the figures 6..2 be set to the left, the root of 6·2 appears below, while if they are set to the right the root of 62 is obtained. One must thus proceed in accordance with the numbers as shown (1...10...100). If the number lies outside the scale range 1 to 100, it should be factorised by hundreds to bring the significant figures within these limits.

Example: $\sqrt{1922} = \sqrt{100 \times 19{\cdot}22} = 10 \times \sqrt{19{\cdot}22} = 10 \times 4{\cdot}38 = 43{\cdot}8$

$\sqrt{0{\cdot}000\,071} = \sqrt{71 \div 1\,000\,000} = \sqrt{71} \div 1\,000 = 8{\cdot}43 \div 1000 = 0{\cdot}008\,43.$

Cubes And Cube Roots

Scale **K** is graduated in the ratio 1:3. In passing over from Scale **D** to **K** the number is raised to the **third power,** while passing from **K** to **D** gives the **cube root,** as shown in Fig. 9. When setting the number for a cube root on Scale **K** it is necessary to watch the values 1 . . . 10 . . . 100 . . . 1000.

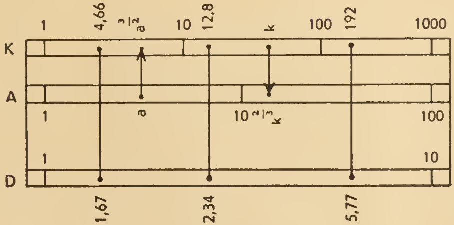


Fig. 9

a^3 $\sqrt[3]{a}$

$a^{\frac{3}{2}}$ $k^{\frac{2}{3}}$

If the number does not lie within the scale range 1 . . . 1000, it must be factorised by thousands to bring it within these limits.

Example: $\sqrt[3]{1\,260\,000} = \sqrt[3]{1000^2 \cdot 1,26} = 10^2 \cdot \sqrt[3]{1,26} = 100 \cdot 1,08 = 108$

$\sqrt[3]{0,32} = \sqrt[3]{320 : 1000} = \sqrt[3]{320} : 10 = 6,84 : 10 = 0,684$

If the cube scale be employed with Scale **A**, powers having the exponents $\frac{3}{2}$ and $\frac{2}{3}$ may be found (Fig. 9).

The Pythagorean Scale

This scale represents the function $y = \sqrt{1 - x^2}$. It is employed in combination with Scale D (= x), the latter having the range of values 0·1 to 1.

Examples: $x = 0{\cdot}8 \quad y = 0{\cdot}6$ (Fig. 10)
Sin $\alpha = 0{\cdot}134$, cos $\alpha = 0{\cdot}991$

$\boxed{\sqrt{1\text{-}x^2}}$

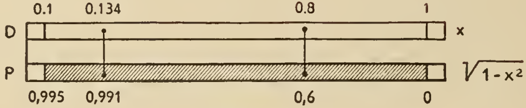


Fig. 10

Example:

Determine the effective current and wattless current of a circuit which absorbs 35 A at 220 V.
$\cos \varphi = 0{\cdot}8$
$I_e = I \times \cos \varphi = 35 \times 0{\cdot}8 = 28$ (A)
$I_w = I \times \sin \varphi = 35 \times 0{\cdot}6 = 21$ (A)

The Mantissa Scale

This scale is used with Scale D for reading **common logarithms,** and may be used in place of a **three-figure table.** Naturally, it only gives the mantissae, the characteristic being found in the usual way.

$\boxed{\log\ x}$

Example: Log 52 = 1·716 (Fig. 11).
Log x = 3·574 x = 3750.

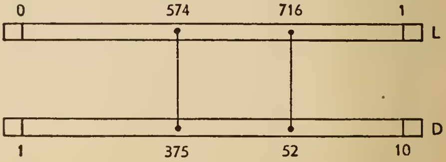


Fig. 11

The Cursor

The three lines on the cursor can be employed as a scale. When the short right-hand line is set to a **diameter** on C or D, the centre line will give the **area** on B or A respectively (Fig. 12); and when the right-hand line is on any given **horse-power,** the left-hand line indicates the corresponding **kilowatts.**

New Cursor

The cursors of the slide rules system DARMSTADT are from now on provided additionally with the mark $\frac{\pi}{4}$ adjacent to the main hairline on the upper left-hand side of the cursor and on the upper right-hand side with a supplementary HP-mark adjacent to main hairline.
Calculating the weight of round bar-iron in kg/m: Set the lower right-hand cursor line on the diameter, e. g. 4,3 cm and read under the $\frac{\pi}{4}$ mark the weight per meter: 11,4 kg. To convert HP into KW and vice versa one sets the upper right-hand cursor line on any given HP-value and reads then under the main hairline on A the corresponding KW-value (and vice versa).

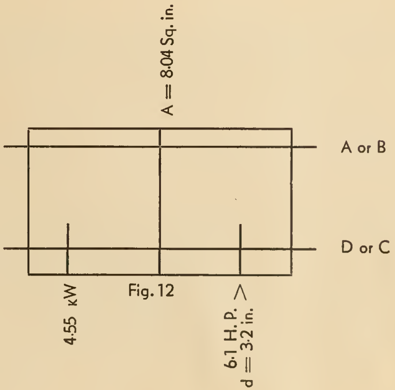


Fig. 12

HP — kw

d — A

The Trigonometrical Scales

Use of the Scales as Tables

Reading the sin-cos table from left to right, with the **Black Numbers**, we obtain a **Sine Table** on Scale D.

With large angles the reading becomes uncertain; in this case it is more accurate if the red numbers are used and read on Scale P. In Fig. 13 sin 76° is given as 0·97 on Scale D, and as 0·9703 on P.

$\boxed{\sin a}$

$\boxed{\cos a}$

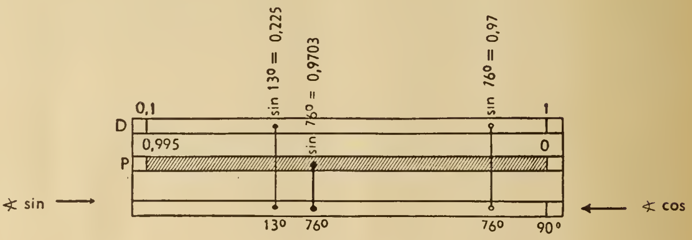


Fig. 13

Reading the **Red Numbers** from right to left, we obtain a **Cosine Table** on D.

With small angles the reading is not exact: it is more accurate if read on Scale P with the black numbers. In Fig. 14 cos 11° is shown on D as 0·982, and more accurately as 0·9816 on P.

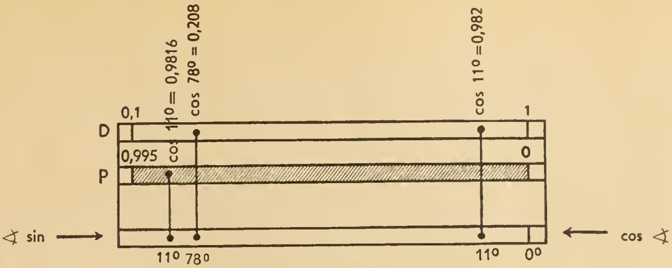


Fig. 14

sin a

cos a

Reading **black numbers** on the tan-cot scale from left to right, we obtain a **tangent table** on Scale D. Reading **red numbers** from right to left, we obtain a **cotangent table** on D.

It would appear as if only tangents of angles below 45° and cotangents of angles over 45° can be read. But as tangent and cotangent values are reciprocal, the use of Scale CI permits all values to be read as shown in the examples of Fig. 15. The procedure is summarised in the following:

Tangents	under 45° over 45°	black numbers and D or C red numbers and CI
Cotangents	under 45° over 45°	black numbers and CI red numbers and D or C

Tan a

Cot a

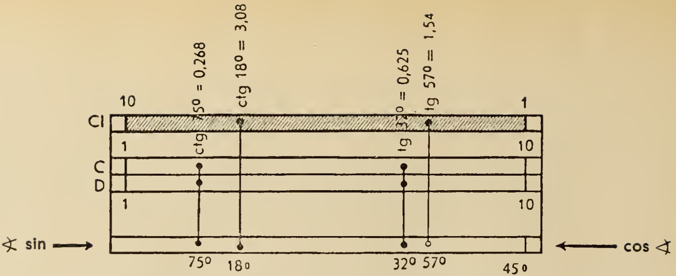


Fig. 15

The trigonometrical functions can be read in this way down to 5°·7. Then, sin 5°·7 ≈ tan 5°·7 ≈ 0·1. The following relationship can be used for still smaller angles: sin α ≈ tan α ≈ arc α ≈ 0·01745 α°.

The mark ϱ has been placed at 1-7-4-5 on the C and D scales; it is employed as shown in Fig. 16. For instance, sin 3° ≈ tan 3° ≈ arc 3° ≈ 0·0524. The error is less than 0·25%.

(The symbol ≈ means "approximately equals".)

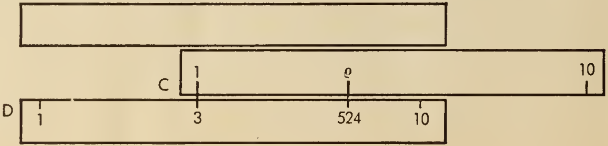


Fig. 16

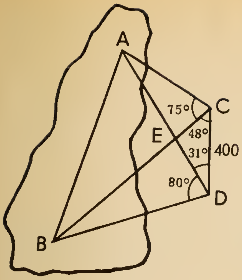


Fig. 17

Example: Find the distance between two points A and B, lying in a moor and therefore inaccessible. Outside the moor the distance CD = 400 yds. and the angles marked in the accompanying diagram are measured. (Fig. 17)

Calculation of I. EA:

$$EA = \frac{EC \times \sin 75^0}{\sin 26^0} = \frac{400 \times \sin 31^0 \times \sin 75^0}{\sin 101^0 \times \sin 26^0} = 462;$$

Calculation of II. EB:

$$EB = \frac{ED \times \sin 80^0}{\sin 21^0} = \frac{400 \times \sin 48^0 \times \sin 80^0}{\sin 101^0 \times \sin 21^0} = 833;$$

Calculation of III. AB:

$AB^2 = EA^2 + EB^2 - 2 \times EA \times EB \times \cos 101^0 = EA^2 + EB^2 + 2 \times EA \times EB \times \cos 79^0 = 213{,}444 + 693{,}889 + 769{,}692 \times 0{\cdot}191 = 907{,}333 + 147{,}011 = 1{,}054{,}344;$

$AB = \sqrt{1{,}054{,}344}$; **AB = 1,027 yds.**

Calculation with Trigonometrical Scales

If it is desired to convert the sine of an angle to the cosine (or vice versa), the angle need not be read. On Scales D and P these pairs of values are shown one under the other. In converting from tangent to cotangent the reading of the angles is likewise dispensed with, as the corresponding values appear one under the other on C and CI. It is only when converting sines or cosines to tangents or cotangents that the angle need be read. In converting sines and cosines to tangents and cotangents it is of advantage to adopt the form

$$\tan x = \frac{\sin x}{\sqrt{1 - \sin^2 x}} \quad \text{and} \quad \cot x = \frac{\cos x}{\sqrt{1 - \cos^2 x}}$$

Example: Given the actual efficiency 32 kW and cos φ = 0·81.
Find the wattless efficiency.

Solution: $\cot \varphi = \frac{\cos \varphi}{\sqrt{1-\cos^2\varphi}}$. Set the cursor line over D 81 ($\cos \varphi = 0{\cdot}81$) and read on the P scale 0·587 ($\sqrt{1-\cos^2 \varphi}$), then set C 587 over D 81 and read above C 1 the answer $\cot \varphi = 1{\cdot}38$ and over D 10 $\tan \varphi = 0{\cdot}724$.

Now set the cursor line over D 32 and read on the C scale the result 23·2 kVA wattless efficiency.

As the functions can be found either on D or on CI, further calculation (multiplication and division) can in many cases be carried out immediately. It is only when the value is read on P that it has to be transferred to the main scales.

The Log-Log-Scale (Exponential Scale) LL_1, LL_2, LL_3

This scale has manifold applications, but only the most important methods of calculating are given here. The following rule will be found helpful:

For a **single calculation** use the **slide** in **normal** position.

For a **series** of problems **invert** the **slide.**

When the slide is inverted the three sections of the log-log scale move between Scales A and D.

There is a tenth power relationship between each pair of adjacent sections of the log-log scale, which makes the reading of tenth roots and tenth powers extremely easy.

$\boxed{a^{10}}$

$\boxed{\sqrt[10]{a}}$

Example:	$1{\cdot}204^{10} = 6{\cdot}4$,	reading	between	2nd	and	3rd	sections
	$1{\cdot}035^{10} = 1{\cdot}41$,	„	„	1st	„	2nd	„
	$\sqrt[10]{75} = 1{\cdot}54$	„	„	3rd	„	2nd	„
	$\sqrt[10]{1{\cdot}248} = 1{\cdot}0224$	„	„	2nd	„	1st	„

These examples show that, with the log-log scale, the position of the decimal point is definitely fixed.

Powers of e

The **exponents** must be set **on** Scale **D.** If they are used in combination with the **lowest** section of the log-log scale, the graduations on D must be read as **1 to 10**; with the **middle** section they must be read as **0·1 to 1**; and with the **upper** section as **0·01 to 0·1.**

The examples given below for calculations with the slide in normal position cannot be solved with the pocket slide rule 67/54 R.

Normal-Slide	Inverted Slide.
Example: $e^{1·61} = 5.$	
Set 1·61 on C to one end of D (say, to 1). Then, turn the rule over and read the answer, 5, on the lowest section under the left-hand index line.	Set the cursor line to 1·61 on D and read the answer, 5, on the lowest log-log section.
Example: $e^{0·61} = 1·84.$	
Set 61 on C (which has to be taken as 0·61) over either end of D (say, over 10). Turn the rule over and, at the right-hand end now, read 1·84 on the middle section.	Set the cursor line over 61 on D (0·61) and read 1·84 on the middle section.
Example: $e^{0·029} = 1·0294.$	
Set 29 on C (which is now 0·029) over either end of D (say, over 1), turn the rule over and read 1·0294, under the index line, on the upper section of the log-log scale.	Set the cursor over 29 (0·029) on D and read 1·0294 on the upper section.

If the power exponent is negative, adopt the form $e^{-n} = \frac{1}{e^n}$, first calculating with the positive n and then finding the reciprocal.

Example: Calculate the elastic force in the running on band of a band brake, of which the band passes round the drum twice.

Solution: $T_{running\ off} = 22$ lbs; $\alpha = 2 \times 360^\circ = \text{arc}\ 4\pi = 12{\cdot}56$;
Coefficient of friction $\mu = 0{\cdot}18$;
$T_{running\ on} = T_{running\ off} \times e^{\mu \times \alpha} = 22 \times e^{2{\cdot}26} = 22 \times 9{\cdot}60 = 211{\cdot}2$ lbs.

$\sqrt[n]{e}$

Roots of e

Example: $\sqrt[4]{e} = e^{\frac{1}{4}} = e^{0{\cdot}25} = \mathbf{1{\cdot}284}$.

If the exponent of the root be changed to a power exponent, as in the above example, the solution is as in the foregoing. The conversion of the exponents is read from the reciprocal scales. It is, however, possible to find the root directly by means of Scale CI.

Normal-Slide

Set 4 on CI over either end (1, for instance) of D. Turn the rule over and read the answer, 1·284, under the left-hand index line on the middle section of the log-log scale.

Inverted Slide.

Set 4 on CI under one of the index lines at the back of the rule (at the left-hand end, for instance). Then the answer 1·284, will be found on the middle section over 1 on D.

Hyperbolic Logarithms

Hyperbolic logarithms are found by reading from the log-log scale to scale D or C.

$\log_e a$

Example: $\text{Log}_e\ 25 = 3{\cdot}22$.

Normal-Slide

Draw the slide to the right until 25 (on the lowest log-log section) appears under the index line. Turn the rule over and read 3-2-2 on C over 10 on D. As the lowest section was used, $\log_e 25 = 3{\cdot}22$.
With the slide to the left the reading is in exactly the same manner.

Inverted Slide.

Set the cursor line on 25 on the lowest log-log section and read log 25 = 3·22 on D.
With this setting we obtain a table of hyperbolic logarithms. There is no movement of the slide.

Example: $\log_e 1{\cdot}31 = 0{\cdot}27$.

Normal-Slide

Draw the slide to the left until 1·31 (on the middle section) appears under the index line. Turn the rule over and read the numbers 2-7 on C over 1 on D. Being on the middle section, it must be read as 0·27.

Inverted Slide.

Set the cursor line over 1·31 on the middle section and read the numbers 2-7 on D. This means 0·27, since it is on the middle section.

Proceed in the same way when reading to the right.

Example: $\log_e 1{\cdot}0145 = 0{\cdot}0144$.

The procedure is exactly as before; the numbers 1-4-4 read on D, must be taken as 0·0144, as the uppermost section was used in setting.

Powers With Fractional Exponents

Example: $3{\cdot}75^{2{\cdot}96} = 50$

$\boxed{a^n}$

Normal-Slide

Bring 3·75 on the lowest section to the right-hand index line (Fig. 18a). Move the cursor over 1 on C (Fig. 18b) and bring 296 on C under the cursor line (Fig. 18c). Turn the rule over and read the answer, 50, under the index line (Fig. 18d)

Inverted Slide.

Set 3·75 (on the lowest log-log section) over 1 on D, move the cursor line over 296 on D and read the answer, 50, directly above (Fig. 19).

a^n

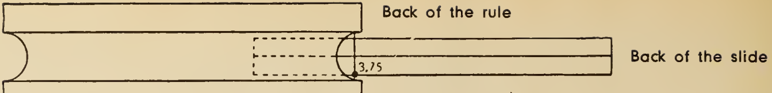


Fig. 18 a

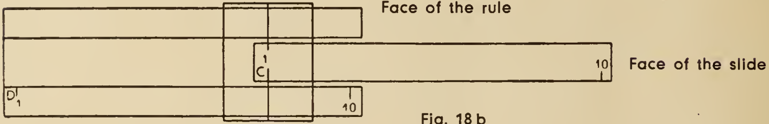


Fig. 18 b

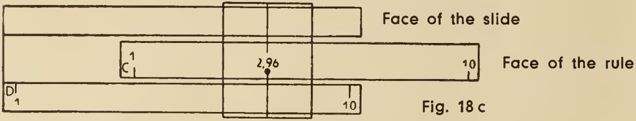


Fig. 18 c

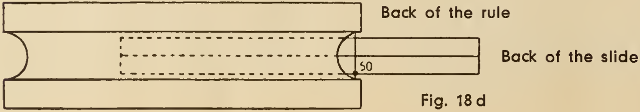


Fig. 18 d

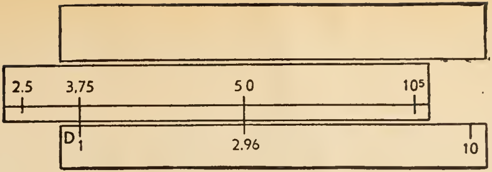


Fig. 19

Example: $1{\cdot}89^{6{\cdot}05} = 47{\cdot}1$.

Normal-Slide

Draw the slide to the right until 1·89 appears under the index line. Put the cursor over 1 on C and bring 605 on C under it. Turn the rule over and read the answer, 47·1, under the left-hand index line. In finding this power, we passed from the middle section to the lowest.

Inverted Slide.

Set 1·89 over 10 on D, using the cursor. Move the cursor over 605 on D and read the answer, 47·1, above it. By this method, the change over from the middle section to the lowest is more noticeable.

Example: $1{\cdot}0525^{29{\cdot}4} = 4{\cdot}5$.

Draw the slide to the right until 1·0525 appears under the index line. Put the cursor over 1 on C and bring 294 on C under it. Turn the rule over and read the answer, 4·5, under the left-hand index-line.

Set 1·0525 over 10 on D, using the cursor. Move the cursor over 294 on D and read the answer, 4·5, above it on the log-log scale.

In the last example, we passed from the highest to the lowest section of the log-log scale. Had the exponent been 2·94, the change over would have been to the middle section (Answer = 1·1623).

There should be no difficulty in selecting the answer on the sections of the log-log scale, as it can be estimated easily.

Roots With Fractional Exponents

When the root exponent is changed to a power exponent by means of the reciprocal scale, the problem is solved as above. It is possible, however, to obtain the answer without the conversion.

$\sqrt[n]{a}$

Example: $\sqrt[4{\cdot}4]{23} = 2{\cdot}04.$

N o r m a l - S l i d e

Draw the slide to the right until 23 appears under the index line. Put the cursor line over 10 on the reciprocal scale CI and bring 44 on CI under it. Turn the rule over and read the answer, 2·04, under the right-hand index line.

I n v e r t e d S l i d e.

Set 23 on the log-log scale (lower section) over 4·4 on D and read the answer, 2·04 (central section) over D 10.

Example: $\sqrt[2{\cdot}08]{1{\cdot}0268} = 1{\cdot}0128.$

In this case, draw the slide to the left until 1·0268 appears under the index line. Put the cursor line over 1 on CI and bring 208 on CI to it. Turn the rule over and read the answer under the left-hand index line.

By means of the cursor line set 1·0268 on the log-log scale (upper section) over D 2·08 and read the answer 1·0128 over D 1.

These brief instructions only indicate the fundamental calculations which can be carried out with the Calculating Rule. For special study **the guide issued in book form** (1/701 e.) is recommended; this contains **numerous examples, with figures,** furnishing an excellent introduction to the practical application of the Calculating Rule.

CASTELL Precision Slide Rules

are the result of many years experience, a culmination of the skilled workmanship of men with long training; the rules are unsurpassed for precision and should be handled with care.

Slide Rules of Specialised Wood Construction

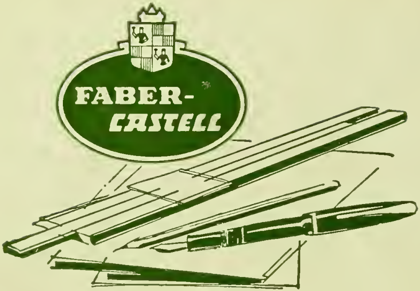
are impervious to climate; they should nevertheless be protected from any considerable temperature fluctuations and from humidity. The resilient base gives the Slide Rule great elasticity, making it easier to move the slide and adjust its mobility. There are metal inserts ensuring exceptional stability in the basic structure of the Slide Rule and preventing deformation by climatic influences.

Slide Rules of Geroplast

are absolutely "climate-proof", as well as heat-proof and damp-proof, and they stand up to the effects of the majority of chemicals. Geroplast Slide Rules should not, however, be allowed to come in contact with corrosive liquids or strong solvents, as these — even if the material itself remains unaffected — are at any rate liable to cause the colour of the graduation marks to deteriorate.

The following applies both to Wood Slide Rules and to Geroplast Rules:

To preserve the legibility of the graduations, the facial scales and the cursor should be protected from dust and scratches, and cleaned at frequent intervals with the special CASTELL cleaning agent No. 211 (liquid) or No. 212 (in paste form).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